Name:

Math 10a Quiz 7
October 23, 2013

1. (2 points) Does the following series converge or diverge? Justify your answer.

$$\sum_{m=1}^{\infty}\left(\frac{5m^2-10}{2m^2+m}\right)^m$$

Root test:

$$\lim_{m\to\infty}\sqrt[m]{\left(\frac{5m^2-10}{2m^2+m}\right)^m}=\lim_{m\to\infty}\frac{5m^2-10}{2m^2+m}=\frac{5}{2}.$$

Since this is bigger than 1, the series diverges.

2. (4 points) Let f be a function that $f(0)=0$ and $f^{(k)}(0)=3^k(k-1)!$ for each $k\geq 1$. What is the largest open interval centered at 0 on which the Taylor series for f centered at 0 converges? Justify your answer.

The Taylor series centered at 0 is

$$\sum_{k=0}^{\infty}f^{(k)}\frac{x^k}{k!}=0+\sum_{k=1}^{\infty}3^k(k-1)!\frac{x^k}{k!}=\sum_{k=1}^{\infty}\frac{3^k}{k}x^k.$$

Since

$$\lim_{k\to\infty}\left|\frac{\frac{3^{k+1}x^{k+1}}{k+1}}{\frac{3^kx^k}{k}}\right|=\lim_{k\to\infty}\left|\frac{3xk}{k+1}\right|=3|x|$$

the ratio test says that the series converges for $3|x|<1$, i.e. for $|x|<\frac{1}{3}$. This is the interval $(-1/3,1/3)$. This is the largest open interval on which it converges because the ratio test says that it diverges for $|x|>\frac{1}{3}$.

3. For this problem, consider the power series $1 + x + x^2 + \cdots = \sum_{k=0}^{\infty} x^k$.

(a) (1 point) For what values of x does the power series converge? (you do not need to justify your answer)

$-1 < x < 1$. This is the geometric series.

(b) (1 point) When the series does converge, what value does it converge to? Write your answer as a rational function of x.

$$\frac{1}{1-x}.$$

This is a formula you learned in lecture.

(c) (2 points) Write down a power series that converges to

$$\frac{1}{(1-x)^2}$$

for $|x| < 1$.

From the previous problem we have

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$$

for $|x| < 1$, so deriving both sides we get

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \cdots = \sum_{k=0}^{\infty} (k+1)x^k.$$